# Impingement of a Two-Dimensional Supersonic Jet upon a Normal Ground Surface

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**A brief report is presented of experimental work on rectangular supersonic jets of nominal Mach no. 1.4 under freely discharging conditions and also impinging upon a normal ground surface. The mixing region and the fully developed region of the free jet are investigated and show good agreement with other authors' work. The pressure distribution on the ground surface under the impinging jet is correlated for various nozzle heights above the ground on the basis of a dimensional argument. Finally, some results of a numerical prediction for the shock-wave shape in the impingement region are also presented.**

## Nomenclature

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| $x$ | = distance from the nozzle in the direction of flow |
| $y$ | = distance from the jet axis measured perpendicularly |
| $x', y'$ | = coordinates $x$ and $y$ in the jet mixing region |
| $x_0$ | = origin for fully developed jet flow |
| $x_s$ | = shock-wave coordinate measured from the ground surface |
| $b$ | = nozzle exit height |
| $h$ | = distance from nozzle exit to ground surface |
| $A_N$ | = nozzle aspect ratio (width/height) |
| $U$ | = mean velocity |
| $M$ | = Mach no. |
| $M_0\,(U_0)$ | = maximum or jet center-line Mach no. (velocity) |
| $M_j$ | = nozzle exit Mach no. |
| $M_\infty$ | = equivalent freestream Mach no. |
| $\sigma$ | = free constant in the jet mixing region |
| $\zeta$ | $= \sigma y'/x'$ |
| $\delta$ | = jet width for fully developed jet |
| $\eta$ | $= y/\delta$ |
| A, B | = constants in Sec. II. |
| $p$ | = pressure (absolute) |
| $p_0$ | = nozzle isentropic stagnation pressure |
| $R_j$ | = jet pitot pressure (gage) |
| $R$ | = ground surface pressure (gage) |
| $J$ | = isentropic jet thrust |
| $F$ | = force on the ground surface |

## I. Introduction

In recent years V/STOL aerodynamics studies have aroused an interest in the fluid mechanics of the impact of a jet upon a plane surface. This paper reports briefly on some parts of an investigation[1] into the impingement of a two-dimensional supersonic jet upon a normal ground surface. The general features of the flow studied and the notation adopted are shown in Fig. 1.

Previous work in this field has been concentrated on three areas: jet development in free air for high-speed and low-speed jets, jet impingement with low-speed jets at large distances from the ground surface, and high-speed jet impingement with the jet traveling only a small distance before impinging upon the ground. The present work brings these three areas together in one investigation that covers the entire field. As a result, work in inviscid supersonic flows is reported, together with turbulent mixing flow investigations. This report progresses from an investigation of the free-jet characteristics through ground surface pressure distributions and forces to stand-off shock wave shapes and locations.

The knowledge of the development of a free jet is clearly a prerequisite to the study of jet impingement upon surfaces. In these terms the important parameters in the jet development are the rate at which the flow velocity decays and the rate at which the jet spreads. Figure 1 shows that a correctly expanded jet develops through an area where the uniform jet-exit flow is surrounded by, and eventually absorbed by, a mixing region to form the fully developed turbulent jet. Previous investigators have performed studies in the mixing region of two-dimensional supersonic jets (Bershader and Pai,[2] and Gooderum et al.[3]) or the fully developed regions of low-speed jets (Bradbury[4,5]) with one notable investigation of a correctly expanded axially symmetrical supersonic jet (Johannesen[6]). Section III presents the results for the characteristics of a two-dimensional jet covering both the mixing region and the fully developed region.

The free-jet characteristics establish a basis for the comparison of ground surface pressure distributions; these are discussed in Sec. IV. An argument advanced by Bradbury[7] is used to correlate the ground surface pressure distributions at various separation distances between the nozzle exit and the ground surface. It also is shown that results from Cartwright and Russell[8] and Milne-Thomson[9] may be correlated with the current results. In Sec. V the experimental results are reported for the variation of the force on the ground surface with the distance between the surface and the nozzle exit. It is noted in this section that, because the nozzle was mounted in a plane surface parallel to the ground surface, the force on the ground could in some circumstances oppose the direction of the jet efflux.

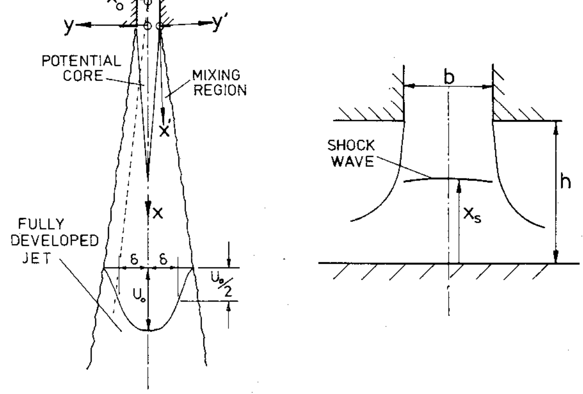


**Fig. 1 General features of the flowfield.**

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interations; Shock Waves and Detonations.

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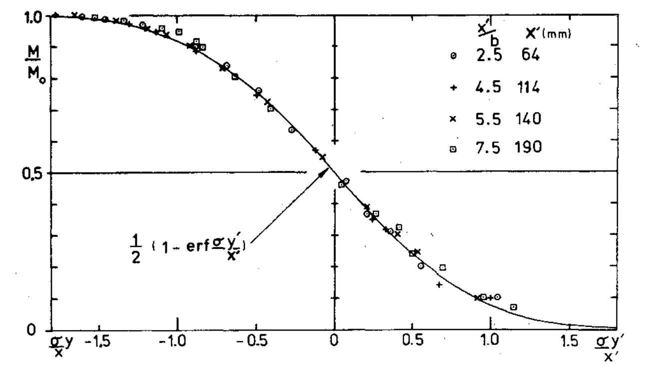

Fig. 2 Mach no. distribution in the mixing region of the free jet. $M_j = 1.4$, $A_N = 4$, $\sigma = 17.0$, $b = 25$ mm.

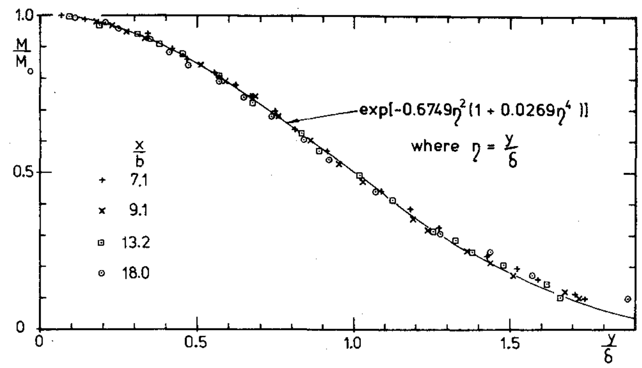

Fig. 3 Mach no. distribution in the fully developed region of the free jet. $M_j = 1.4$, $A_N = 8$, $b = 12.5$ mm.

Section VI is a description of the results of the application of a method to predict the shape of the "stand-off" shock wave. Various authors have attempted to predict the parameters in the mixed supersonic subsonic impingement region and to compare the predicted results with experimental data. Gummer and Hunt[10] used a polynomial approximation of the method of integral relations to predict the shape of the shock wave for an axially symmetrical supersonic jet and obtained encouraging results. More recently Sinha et al.[11] used a time-dependent method to predict the entire flowfield with great success for their chosen case. The results reported here were obtained from a linear approximation for the method of integral relations similar to the method used by Gummer and Hunt[10] but applied here to the equations for the two-dimensional impingement problem. The predicted results are compared with experimental data for the shock-wave shape obtained by using shadowgraph and schlieren techniques.

## II. Experimental Apparatus

The apparatus was arranged in a system similar to a blowdown wind tunnel and operated from a compressed air storage vessel. The nozzle and ground plane were confined between parallel walls 0.1 m apart. The jet was produced through the nozzle by using two flexible plate liners, and the geometry of the linears also provided for changes in the nozzle exit height. Two nozzle heights were used in the current tests, 25 mm and 12.5 mm giving jet aspect ratios of 4 and 8, respectively. The nozzle exit was arranged so that the jet emerged perpendicularly from a plane surface which extended approximately 0.3 m either side of the jet centerline. The ground surface was perpendicular to the jet axis and for the tests reported here also extended 0.3 m on either side of the jet axis. Pressure tappings were made on the ground surface and the nozzle plane, and also along the nozzle at appropriate points. The walls confining the apparatus could be fitted with windows in order that shadowgraph and schlieren flow visualization studies could be made of the flow through the nozzle and the impingement region. The apparatus and the experimental procedures are described in greater detail by Pollard.[1]

## III. The Free Jet

The general features of a correctly expanded free jet are shown in Fig. 1. This section describes the results of pressure traverses across freely discharging jets of aspect ratio 4 and 8 under correctly expanded conditions for a nominal Mach no. of 1.4. The pressure traverses were performed across the jet within the range of distances from the jet exit, 0 to 18 jet exit heights downstream. The results were found to agree with the generally accepted patterns of a mixing region around a "potential" core for about 7 exit heights beyond the nozzle exit followed thereafter by a fully developed profile.

Within the mixing region the Mach no. distribution could be well-matched by the usual error function expression.

$$M/M_0 = \tfrac{1}{2}\,(1-\mathrm{erf}\,\zeta) \tag{1}$$

where $\zeta = \sigma\, y'/x'$. The matching was achieved by adopting appropriate values for the free constant $\sigma$. The values chosen to give the best fit were 17.0 and 16.9 for the jets of aspect ratio 4 and 8, respectively. These values compare favorably with the value obtained by Bershader and Pai[1] on a two-dimensional jet ($M = 1.7$). Work on axisymmetric jets of Mach no. 1.4 by Johannesen indicated rather lower values for $\sigma$ (13.7 and 11.9) which were closer to the established results for low-speed jets ($11 < \sigma < 12$). These results taken together would seem to suggest a possibility that for very high velocities there is some difference in the rate of mixing for an axisymmetric jet from that experienced by a two-dimensional one. Figure 2 shows the Mach no. distributions in the mixing region for the jet of aspect ratio 4, the results for the narrower jet exhibited a similar degree of agreement with the error function profile.

The fully developed region of the jets followed the expected pattern and showed fair agreement with the expressions suggested by Bradbury.[5] The general forms for a two-dimensional jet issuing into still air that were suggested were

$$\frac{M}{M_j} = \frac{A}{(x - x_0/b)^{1/2}}\ ; \ A \simeq 2.5$$

$$\delta/b = B(x - x_0/b)\ ; \ B \simeq 0.109$$

$$U/U_0 = \exp[-0.6749\eta^2(1 + 0.0269\eta^4)] \tag{2}$$

where $\eta = y/\delta$. The current work yielded values of 2.61 and 0.095 for the constants A and B and 2.85 and 0.08 for the jets of aspect ratio 4 and 8, respectively. Both jets show good agreement with the Mach no. profile as is shown in Fig. 3 for the aspect ratio 8 jet. It also was established that small amounts of under-expansion of the nozzle flow resulted in some change in the position of the effective origin $x_0$ for the jet but little change in the other parameters of the fully developed jet.

## IV. Ground Surface Pressure Distributions

Bradbury[7] has suggested a simple argument for correlating data obtained in the impact region of jets impinging upon a normal ground. The effect of the argument was a suggestion that, provided mean velocity similarity has been established within the jet, the pressure distributions upon the ground may be correlated upon the basis of the free-jet dynamic pressure and the free-jet width at the location of the ground. It was also suggested that the argument would be valid for high-speed jets although no experimental data was provided. The work reported herein confirms the suggestion. However, there

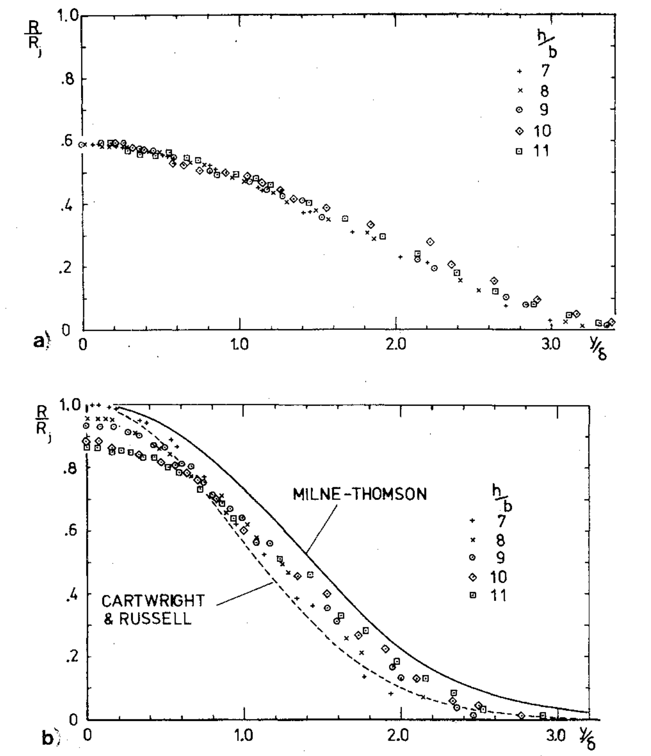

Fig. 4 a) Ground surface pressure distribution, free jet correlation. $M_j = 1.4, A_N = 4, b = 25$ mm, $x_0 = 0.077b$. b) Ground surface pressure distribution, correlation with modified origin. $M_j = 1.4$, $A_N = 4$, $b = 25$ mm, $x_0 = -3.64b$.

are some modifications in the application of the argument to the higher jet velocities.

Initially a correlation was attempted on the basis of the pitot pressure and jet width in the free jet at the location of the ground surface. This led to a satisfactory collapse of the data as shown in Fig. 4a, but a maximum ground surface pressure of 60% free-jet pitot pressure. Figure 4b shows the results of a second procedure using the free jet rate of spread and rate of decay characteristics but with a modified origin $x_0$. The position of the new origin was chosen to provide a matched pitot pressure at the smallest appropriate separation distance- $h/b = 7$. The collapse of the data is again satisfactory, but the need for the second procedure points to one of the difficulties in this method of correlation. The problem arises because the effective origin of the jet is difficult to determine without experiments. Fortunately this is only a problem when the ground surface is near to the nozzle as it was in the present work. The change in the position of the origin for the two procedures above was from $x_0/b = +0.077$ to $x_0/b = -3.64$. With large distances between the nozzle and the ground surface this change could be neglected but with the relatively small distances reported here it is of considerable importance. Also shown in Fig. 4b are two curves from other sources. Milne-Thomson[9] quotes a potential flow solution for a uniform two-dimensional jet impinging upon an infinite ground surface. By adopting a characteristic turbulent jet profile, an equivalent jet width $\delta$ having equal thrust to the uniform jet was determined. The results then could be plotted as shown in Fig. 4b. Cartwright and Russell[8] presented experimental results for a low-speed turbulent slot jet; by assuming a typical rate of spread for their jet, the second curve on Fig. 4b was obtained. All of these results show a gratifying agreement and further justify the use of the criteria suggested by Bradbury.

## V. Ground Surface Forces

It is well known that, when a low-speed jet issues from a plane surface and impinges normally upon another plane surface in close proximity, there is a position when the two surfaces are mutually attracted. Although "lift loss" has been demonstrated in VTOL aircraft, it has not been fully accepted that the low-speed attraction also takes place with jets of much higher velocity. Figure 5 shows the variation of load on the ground surface with distance between the nozzle exit plane and the ground surface for the jet arrangement as aspect ratio 4. The ground surface and nozzle exit planes extended for 12 nozzle exit heights on either side of the jet centerline. The force was computed from the results of the closely spaced pressure tapings upon the surface. It can be seen that the net force on the ground plane does change direction and that within a small range a force well in excess of the jet thrust is exerted attracting the ground plane towards the nozzle exit. It was not possible to measure the jet thrust so it was estimated from the nozzle stagnation pressure by assuming an isentropic expansion to ambient pressure at the nozzle exit. A similar computation of the results to produce the force on the nozzle exit plane established that the net force on both planes was within 10% of the jet thrust, thus confirming the accuracy of the work.

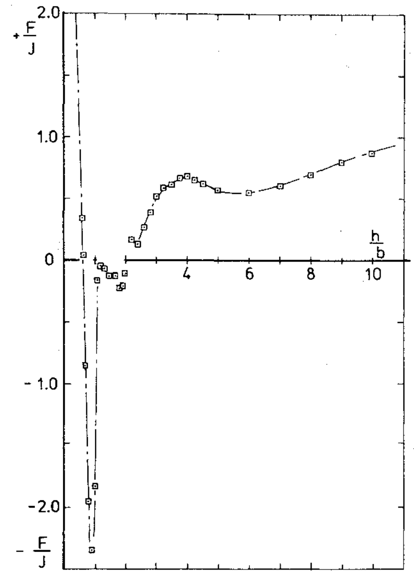

Fig. 5 Force on the ground surface for various heights between the nozzle exit and the ground. Extent of the ground on either side of the centerline = $12b$. $M_j = 1.4, A_N = 4, b = 25$ mm.

## VI. Supersonic and Impingement Structure

Although predictions of the flow are not possible with relatively large distances between the nozzle and the ground surface, some predictions are possible for the supersonic flow at smaller distances. Gummer and Hunt[10] showed that the method of Polynomial Approximations and Integral Relations could be used with some success to predict the shock-wave shape and part of the pressure distributions for an axially symmetric supersonic jet. A similar method has been applied to the two-dimensional case, and comparisons with experimental data showed similar success. The basic equations for this numerical method are not quoted here, but they are readily available in Refs. 1 and 10.

One of the main problems in the computed solution is the determination of its scale. Gummer and Hunt[10] suggested two possible means of scaling; a) a scale based upon the position of the sonic point on the shock-wave, and b) a scale based upon the position the position of the sonic point on the ground surface. In the axially symmetric cases investigated by Gummer and Hunt[10] both of these scaling criteria could be used with only small differences between the results. However, within the numerical procedure, a singularity can occur beyond which the computation cannot proceed. When the basic equations are written for the two-dimensional case,

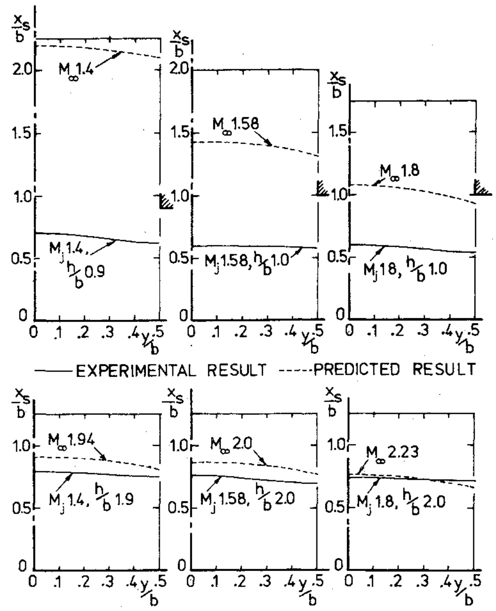

Fig. 6 Shock-wave shape at various Mach numbers, comparison of results.

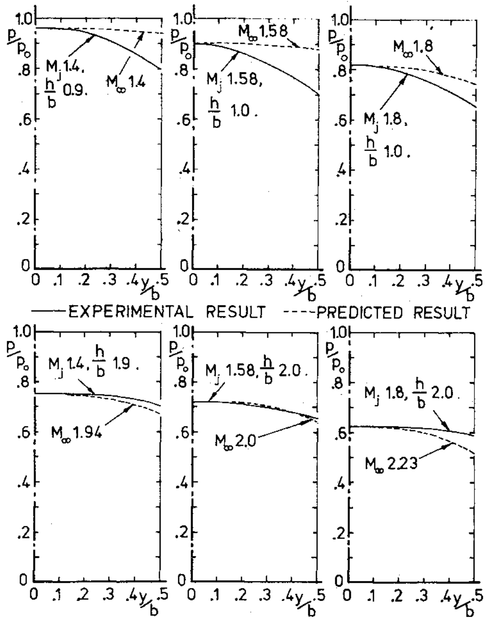

Fig. 7 Pressure distributions on the ground surface in the immediate impingement region; comparison of results.

the computation shows that the sonic point on the shock wave is much closer to the jet axis than the sonic point on the ground surface. In particular, within the range of Mach nos. considered here, the singularity occurs before the ground surface sonic point has been reached. It was only possible, therefore, to scale the solution by positioning the sonic point on the shock wave at the edge of the jet. Calculations were performed for a range of Mach nos. for 1.4 to 2.23 with three nozzle area ratios equivalent to 1.4, 1.58, and 1.8. The higher equivalent Mach nos. in the jet were obtained at ground surface positions where the jet had undergone expansion outside the nozzle, and the shock wave was observed to span a disturbance-free jet. These latter conditions were equivalent to freestream Mach nos. 1.94, 2.00, and 2.23. Figures 6 and 7 show the results for the shock-wave shape and position and the pressure distributions on the ground surface from the calculations and from experimental observations. At the lower Mach nos. the experimental and predicted results differ considerably, but agreement is acceptable at the higher numbers.

The Mach nos. for the experimental work, and hence for the calculations, are low values with respect to the normal use for the Method of Integral Relations. The varying agreement between the predicted and experimental results therefore is not surprising, as Gummer and Hunt[10] also noted. Nevertheless, the work reported here confirms that useful results may be obtained for a two-dimensional jet of aspect 4.

## VII. Conclusions

Some results of an investigation with a two-dimensional supersonic jet of Mach no. 1.4 have been presented. The characteristics of the free jet in both the mixing region and the fully developed region were shown to agree with other authors' work. In addition, within the mixing region it seems that it may be possible that, for very high jet velocities, there is some difference in the rate of mixing for an axisymmetric jet from that in a two-dimensional jet.

The impinging jet has been studied, and the pressure distributions on the ground surface for various distances between the nozzle and the ground have been correlated on the basis of the free jet characteristics. One small limitation to the practical use of the correlation method has been noted. As a final contribution, some results have been presented for a numerical prediction procedure for the shock-wave shape and location.

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